Imperial College London Mathematics School Admissions Test

2025 Sample 1

Mark Scheme

Marking instructions

- Each question in sections A and B scores 2 marks for the correct answer or zero for no answer, the wrong answer or more than one answer.

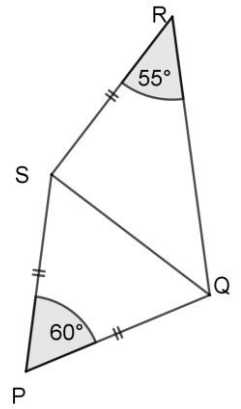
- Questions in section C may be worth 1 or 2 marks for the correct answer (as indicated) or zero for no answer, the wrong answer or more than one answer.

Section A

Number	Solution	Mark	Guidance
1	**C** $1-(9-x)$	**2**	Either 2 or zero for each question on Section A. **Example reasoning** $1-(9-7)=1-2=-1$ The other expressions all give 3 when $x=7$.
2	**A** $m=3, n=13$	**2**	**Example reasoning** $3^3=27$ so $m=3$ $(3\times10^4)^3=27\times10^{12}=2.7\times10^{13}$
3	**D** 29	**2**	**Example reasoning** $3x-5<84$ $3x<89$ $x<29\frac{2}{3}$ So 1, 2, ….28,29 are possible values
4	**E** All of them	**2**	**Example reasoning** In each rectangle, the longer side is 2.1 times the shorter side.
5	**E** $y=2x^3-5x^2+2x-4$	**2**	**Example reasoning** The curve is not a quadratic curve so that rules out A and C. The curve looks as though it goes through (0, -4) For D, when $x=0$, $y=4$ so it can't be D When $x=2$ B: $y=8-4=4$ so B is ruled out.

Number	Solution	Mark	Guidance
6	**D** 1.5	**2**	**Example reasoning** $x = -2$ is a root so $4a - 2 = 6$ $4a = 8$ $a = 2$ $2x^2 + x - 6 = 0$ $(x + 2)(2x - 3) = 0$ $x = -2$ or $x = 1.5$
7	**B** 2023 is a term in both sequences and occurs earlier in the first sequence.	**2**	**Example reasoning** The mth term of the second sequence is $3m + 7$ $3m + 7 = 2023$ $3m = 2016$ 2016 is a multiple of 3 so 2023 is in the second sequence. The nth term of the first sequence is $4n - 1$ $4n - 1 = 2023$ $4n = 2024$ 2024 is a multiple of 4 so 2023 is in the first sequence. The first sequence goes up in 4s so will soon overtake the second and so 2023 will occur earlier in the first sequence

Number	Solution	Mark	Guidance
8	**C** 2	**2**	**Example reasoning** $\frac{6V}{\pi h} = 3a^2 + 3b^2 + h^2$ $\frac{6V}{\pi h} - 3b^2 - h^2 = 3a^2$ $\frac{6V}{3\pi h} - b^2 - \frac{h^2}{3} = a^2$ This leads to the first and second rearrangements which are both correct. To get a formula for b, similar working to the above leads to $\frac{6V}{3\pi h} - a^2 - \frac{h^2}{3} = b^2$ So the third rearrangement is incorrect. Multiplying out the bracket in the original formula gives terms in h and terms in h^3 so it is not possible to rearrange to get a formula for h

Number	Solution	Mark	Guidance
9	**E** 130°	**2**	**Example reasoning**  Add in the line SQ. Triangle PQS is isosceles with one angle 60° so it must be equilateral with all angles 60°. SQ=RS so triangle QSR is isosceles and angle RSQ = 70° Angle PSR = 60°+70°=130°
10	**A** $h = \frac{r\sqrt{7}}{4}$	**2**	**Example reasoning** The circumference of the circle at the base of the cone is $\frac{3}{4} \times 2\pi r$ so the radius of this circle is $\frac{3r}{4}$ The slant height of the cone is r Using Pythagoras' theorem, $h^2 = r^2 - \left(\frac{3r}{4}\right)^2$ $h^2 = \frac{7r^2}{16}$ $h = \frac{r\sqrt{7}}{4}$

Section B

<table>
<tr><th>Number</th><th>Solution</th><th>Mark</th><th>Guidance</th></tr>
<tr><td>11</td><td>D 54</td><td>2</td><td>Either 2 or zero for each question on Section B.
Example reasoning
For every 10 children there is 1 adult
50 children would need 5 adults. This would take up 55 seats.
The 5 seats left can be filled with 4 children and 1 adult.</td></tr>
<tr><td>12</td><td>D<table><tr><th>smallest</th><th></th><th>largest</th></tr><tr><td>90% of 30</td><td>40% of 89</td><td>89% of 41</td></tr></table></td><td>2</td><td>Example reasoning
The three calculations are
$\frac{90\times30}{100}$
$\frac{40\times89}{100}$
$\frac{89\times41}{100}$
The third is clearly bigger than the second.
$90 \times 30 = 90 \times 40 \times \frac{3}{4}$ and $90 \times \frac{3}{4}$ is less than 89 so this is the smallest one</td></tr>
<tr><td>13</td><td>E 77</td><td>2</td><td>Example reasoning
Making 12 fewer large biscuits allows 48 more small ones to be made.
Each large biscuit uses 4 times as much mixture as a small one.
The mixture for 6 large biscuits could make 24 small ones.
24 + 53 = 77</td></tr>
<tr><td>14</td><td>C 18</td><td>2</td><td>Example reasoning
$495 = 55 \times 9$
$110 = 55 \times 2$
The squares are 55 cm by 55 cm and there are 2 of them one way and 9 the other making 18 in all.</td></tr>
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Number	Solution	Mark	Guidance
15	**B** 3	**2**	**Example reasoning** For the first third and fourth set of numbers, the median is £34.50 and the distribution is symmetrical so the mean will also be £34.50.
16	**A** 0	**2**	**Example reasoning** There are more crosses for 40 hours than for any other number of hours so the first statement is true. There are people who seem to be on the horizontal axis, they have zero wages. Some of them work some hours a week so the second statement is true. The third statement is true. People with zero hours worked who also have zero wages would be at the origin and there is a point which seems to be at the origin so the last statement is true.
17	**B** 13 years	**2**	**Example reasoning** The circumference of the earth is $2\pi \times 3959 \approx 6 \times 4000 = 24000$ miles $24000 \div 5 = 4800$ days There are about 400 days in a year $4800 \div 400 = 12$ years There are less than 400 days in a year so it will take a bit more than 12 years
18	**A** 0	**2**	**Example reasoning** The cost in pennies of any combination of 15p and 9p sweets will be divisible by 3. 220 is not divisible by 3. It cannot be done.

Number	Solution	Mark	Guidance
19	**D** 720	**2**	**Example reasoning** If the code starts 0 then the last two digits can be 12 to 98 but not 22, 33 etc and not 20, 30, etc so $$98 - 11 - 7 - 8 = 72$$ There are the same number of codes starting 1, 2, 3 etc so $10 \times 72 = 720$
20	**C** 1.8	**2**	**Example reasoning** The faster lorry is gaining on the slower one at 40 km per hour. Overtaking starts when the front of the faster lorry draws level with the back of the slower lorry and ends when the back of the faster lorry draws level with the front of the slower one. This is 20 metres. Time needed is the time to travel 20 metres at 40 km per hour. 40 km per hour is 40 000 metres per hour. $$\text{Time} = \frac{20}{40000} \text{ hours} = \frac{1}{2000} \text{ hours}$$ 1 hour is 3600 seconds so time is $\frac{3600}{2000} = 1.8 \text{ seconds}$

Section C

Number	Solution	Mark	Guidance
21a	A (2, 0)	**1**	**Example reasoning** $3x + 2y = 6$ $y = 0$, $3x = 6$ so $x = 2$
21b	E (7, 0)	**1**	$x - y = 7$ $y = 0$, $x = 7$
21c	C (4, -3)	**1**	$3x + 2y = 6$ $2x - 2y = 14$ $5x = 20$ $x = 4$, $y = -3$
21d	E 3	**2**	Curve going through P and Q is $y = k(x - 2)(x - 7)$ $x = 4, y = -3$ $-3 = k \times 2 \times -3$ $-3 = -6k$ $k = \frac{1}{2}$ $y = \frac{1}{2}(x - 2)(x - 7) = \frac{1}{2}(x^2 - 9x + 14)$ $a = \frac{1}{2}, b = -\frac{9}{2}, c = 7$ $a + b + c = \frac{1}{2} - \frac{9}{2} + 7 = -4 + 7 = 3$

Number	Solution	Mark	Guidance
			Alternative method $4a + 2b + c = 0$ $49a + 7b + c = 0$ $16a + 4b + c = -3$ Elimination of one variable from two pairs of equations to give two simultaneous equations in two variables $45a + 5b = 0$ $12a + 2b = -3$ $a = \frac{1}{2}, b = -\frac{9}{2}, c = 7$

Number	Solution	Mark	Guidance
21d	C Bharat and Christian only	**2**	$y = \frac{1}{2}(x^2 - 9x + 14) = \frac{1}{2}(x^2 - 9x) + 7$ So the y intercept is $(0, 7)$ and Daniela is correct $= \frac{1}{2}\left(\left(x - \frac{9}{2}\right)^2 - \frac{81}{4}\right) + 7$ $= \frac{1}{2}\left(x - \frac{9}{2}\right)^2 - \frac{81}{8} + 7$ So the line of reflection symmetry is $x = 4.5$ and Anita is correct Christian and Bharat must be incorrect. Check (not necessary): $-\frac{81}{8} + 7 = \frac{56-81}{8} = -\frac{25}{8} < -3$ so Bharat is incorrect From $2y = x - 10$, $x = 2y - 10$ In $y = \frac{1}{2}(x - 2)(x - 7)$ this gives $y = \frac{1}{2}(2y - 12)(2x - 17)$ $y = \frac{1}{2}(4y^2 - 58y + 12 \times 17)$ $y = 2y^2 - 29y + 6 \times 17$ $2y^2 - 30y + 6 \times 17 = 0$ $y^2 - 15y + 3 \times 17 = 0$ $y^2 - 15y + 51 = 0$ This is not equivalent to $(y - k)^2 = 0$ so it does not have one solution.
		[7]	

Number	Solution	Mark	Guidance
22a	C $\frac{3}{8}$	**1**	Either consider outcomes HHH, HHT, HTH, THH, HTT, THT, TTH, TTT. 3 of 8 of these have exactly one head Or HTT, THT, TTH $3\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$
22b	E $\frac{2}{9}$	**2**	Either the biased coin is heads and the other two are tails or fair coin 1 is heads and the other two are tails or fair coin 2 is heads and the other two are tails giving $\frac{1}{4}p+\frac{1}{4}(1-p)+\frac{1}{4}(1-p)=\frac{1}{4}(2-p)$ $\frac{1}{4}(2-p)=\frac{4}{9}$ $2-p=\frac{16}{9}$ $p=2-\frac{16}{9}=\frac{18-16}{9}=\frac{2}{9}$

22c	B I and III only	**2**	Either the biased coin is tails and the other two are heads or the biased coin is heads, fair coin 1 is heads and fair coin 2 is tails or the biased coin is heads, fair coin 1 is tails and fair coin 2 is heads. $\frac{1}{4}(1-p)+\frac{1}{4}p+\frac{1}{4}p=\frac{1}{4}(p+1)$ $\frac{1}{4}(p+1)=\frac{7}{24}$ $p+1=\frac{7}{6}$ $p=\frac{7}{6}-1=\frac{7}{6}-\frac{6}{6}=\frac{1}{6}$ at least two tails is either exactly 1 head or no heads $=\frac{1}{4}\left(2-\frac{1}{6}\right)+\frac{1}{4}\times\frac{5}{6}=\frac{1}{4}\times\left(\frac{12-1}{6}+\frac{5}{6}\right)=\frac{1}{4}\times\frac{16}{6}=\frac{4}{6}=\frac{2}{3}$ So I is correct P(tails on unfair coin)= $1-\frac{1}{6}=\frac{5}{6}$ so II is incorrect For exactly 2 tails either the biased is tails, fair coin 1 is tails and fair coin 2 is heads or biased is tails, fair coin 1 is heads and fair coin 2 is tails or biased is heads and both fair coins are tails giving $\frac{1}{4}\times\frac{5}{6}+\frac{1}{4}\times\frac{5}{6}+\frac{1}{4}\times\frac{1}{6}=\frac{5+5+1}{24}=\frac{11}{24}$ so III is correct P(at least 1 heads) $=1-$ P(TTT) $=1-\frac{1}{4}\times\frac{5}{6}=1-\frac{5}{24}$ $=\frac{24-5}{24}=\frac{19}{24}$ so IV is not correct
		[5]	

Number	Solution	Mark	Guidance
23a	E $\frac{4\sqrt{3}}{3}$.	**1**	$\sin 60° = \frac{2}{s}$ where s is a side of the large hexagon $s = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$
23b	B $\frac{1}{10}$	**2**	The side length of one of the small hexagons is $\frac{1}{2}$ of that of a large hexagon so it's area is $\frac{1}{4}$ of a large hexagon. The shaded area is therefore made up of $4 + 4 + 1 + 1 = 10$ small hexagon areas and so each small hexagon is $\frac{1}{10}$ of the shaded area
23c	D Adam and Dani.	**2**	The length of side $AB = 2 \times 2\tan 30° + 3 \times \frac{2\sqrt{3}}{3} + 2 \times \frac{4\sqrt{3}}{3} = \frac{4\sqrt{3}+6\sqrt{3}+8\sqrt{3}}{3}$ $= 6\sqrt{3}$ Adam $9 \times \frac{2\sqrt{3}}{3} = 6\sqrt{3}$ true Bailey $2\frac{1}{2} \times 4 = 10 \neq 6\sqrt{3}$ false Cara $4 \times \frac{4\sqrt{3}}{3} = \frac{8}{3}\sqrt{3}$ false Dani $\frac{3}{4} \times 6 \times \frac{4\sqrt{3}}{3} = 6\sqrt{3}$ true
23d	A $24\sqrt{3}$	**2**	Area $= 4 \times 6\sqrt{3} = 24\sqrt{3}$
23e			Area of large hexagon $= 6 \times \frac{1}{2} \times \frac{4\sqrt{3}}{3} \times 2 = 8\sqrt{3}$ Area of small hexagon $= \frac{1}{4} \times 8\sqrt{3} = 2\sqrt{3}$ Shaded area $= 20\sqrt{3}$ Fraction of $ABCD = \frac{20\sqrt{3}}{24\sqrt{3}} = \frac{20}{24} = \frac{5}{6}$
		[7]	